Concurrent Design Forces in Structures under Three-Component Orthotropic Seismic Excitation

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According to the model of Penzien and Watabe, the three translational ground motion components on a specific point of the ground are statistically noncorrelated along a well-defined orthogonal system of axes p, w, and v, whose orientation remains reasonably stable over time during the strong motion phase of an earthquake. This orthotropic ground motion is described by three generally independent response spectra S^a, S^b, and S^c, respectively. The paper presents an antiseismic design procedure for structures according to the above seismic motion model. This design includes a) determination of the critical orientation of the seismic input, i.e., the orientation that gives the largest response, b) calculation of the maximum and the minimum values of any response quantity, and c) application of either the Extreme Stress Method or the Extreme Force Method for determining the most unfavorable combinations of several stress resultants (or sectional forces) acting concurrently at a specified section of a structural member. [DOI: 10.1193/1.1463040]

INTRODUCTION

According to the idealization of Penzien and Watabe (1975), the seismic input at the base of any given structural system can be considered as consisting of three simultaneously occurring and statistically noncorrelated components directed along a set of principal axes: a vertical and two orthogonal horizontal excitations. In the initial phase of the ground motion, the orientation of these principal axes is changing with time. However, after a short period of time it becomes practically stable: the major principal axis "*p*" (strong excitation) is horizontal and directed towards the epicenter, the intermediate principal axis "*w*" is in the transverse (orthogonal) direction, and the minor principal axis "*v*" (weak excitation) is vertical. (The chosen notation shall remind of Penzien-Watabe model.) It is clear however, that this model is valid only when near-source effects are absent.

The orthotropic ground motion postulated by the above idealization can be described by three, generally independent, response spectra S^a, S^b, and S^c, respectively. A simplified form of this seismic input model has been already accepted by many modern earthquake design codes. The simplification usually consists of prescribing the same design response spectrum $S^a=S^b$ for both horizontal components (i.e., isotropic excitation) and setting $S^c=(2/3)S^a$ for the vertical one.

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In recent years, progress has been made on the response spectrum method of analysis of structures subjected to the described multidirectional seismic excitation (Gupta and Singh 1977; Gupta and Chu 1977; Smeby and Der Kiureghian 1985; Der Kiureghian 1981; Anastassiadis and Avramidis 1992, 1993; Anastassiadis 1993, Wilson et al. 1995; Lopez and Torres 1996; Anastassiadis and Avramidis 1996; Menun and Der Kiureghian 1998; Lopez et al. 2000, 2001). However, the results of the related research work are not yet widely known and, despite their simplicity in some cases of practical importance, are not yet incorporated either in seismic design codes nor in the professional software packages for structural analysis (Wilson 1996). As an example, clause 3.3.3.1 of the European Seismic Code *EC8/94*, Part 1-2 demands the determination of the most unfavorable orientation of the bi-directional seismic excitation, thus ignoring the fact that for a building subjected to *isotropic* bidirectional seismic input the peak values of all response quantities do not depend on the input's orientation. Furthermore, the spatial combination of response quantities resulting from two (or three) seismic components is still based on the empirical "percentage combination rules" of the 100-30-30 or 100-40-40 type, thus overlooking the exact formulae for the extreme value of a given quantity as well as for the concurrent values of two (or more) quantities (Gupta and Chu 1977, Anastassiadis 1993).

In this paper, the tensorial properties of an arbitrary response quantity of a structure subjected to a general *orthotropic*[1] seismic excitation, first presented by Anastassiadis (1993), are re-derived using a simpler methodology and summarized. From these general relations all relevant special cases, e.g., the bidirectional excitation with *identical* spectral shapes of ratio λ, are easily derived. Since the orientation of the principal axes is unknown, exact relations and formulae are given, which can be used for a straightforward calculation of the *critical* seismic input angle θ_{cr}, i.e., the orientation that gives the largest response, and of the maximum and minimum values of any response quantity. Furthermore, on the basis of the above tensorial properties, two methods are presented (Extreme Stress Method and Extreme Force Method) that can be used for determining the *most unfavorable* combinations of two or more simultaneous stress resultants at any cross section of a structural member in the general case of orthotropic seismic input.

With the aid of the two aforementioned methods, any unfavorable combination of responses needed can be *immediately* calculated for a small number of critical values of the seismic input angle θ. The calculations are based on a few discrete points of the corresponding Gupta-envelope (Gupta and Singh 1977, Anastassiadis 1993) and do not require its explicit determination. In this way, the presented methods take advantage of the *reliability* of the Gupta-envelope and, at the same time, avoid its explicit determination; in the present case of orthotropic seismic excitation, that would require consideration of all values of the seismic input angle θ in the interval $(0, \pi)$. Also avoided here is the use

[1]The terms "isotropic," "anisotropic," and "orthotropic" come from the Greek language and have exactly the same meaning as the Greek words "ισότροπος," "ανισότροπος," and "ορθότροπος." The first of these words means "having the same properties or behavior ('tropos') in any direction," the second means "having different properties or behavior in different directions," and the third means "having different properties or behavior along two directions orthogonal to each other."

of the so-called supreme-envelope (Menun and Der Kiureghian 2000a, b), i.e., the union of all Gupta-envelopes, the determination of which is extremely laborious and must be carried out point by point.

The main disadvantage of any method using envelopes, either for orthotropic or for isotropic seismic input, is not just the difficulty of the envelopes calculation in the 2-D or 3-D response space, but also the more laborious determination of the *contact point* at which the capacity curve or surface encompasses and touches the response envelope. In applying the Gupta-method for the 2-D case, this specific problem can be dealt with either graphically (by plotting the ellipse on the M-N interaction diagram) or analytically (by replacing the ellipse by a *circumscribed octagon* and calculating the reinforcement for each octagon corner; see Leblond 1980, Panetsos and Anastassiadis 1998). Similarly, in the 3-D case, the ellipsoid is replaced by a *circumscribed polyhedron* with 24 corners. It becomes obvious that, apart from any computational difficulties, the final response quantities resulting from the Gupta-method, will always be larger than the exact ones because the octagon or polyhedron corners lie outside the corresponding envelopes.

Greater difficulties arise concerning the supreme-envelope method. The determination of the contact point between the capacity curve or surface and the supreme-envelope requires the use of a special numerical algorithm in an iterative manner, whose convergence to a global or local maximum or minimum cannot be predicted from the outset. (For more details see Menun and Der Kiureghian 2000b, p. 476).

In contrast to the above-mentioned difficulties, which are almost insuperable for practicing engineers, application of the Extreme Stress and Extreme Force Methods is *straightforward* and requires only elementary matrix manipulations that cause no particular problems at all.

The reliability of the Extreme Stress Method is *identical* to that of the original method by Gupta and Singh 1977 (that makes no use of envelope octagons or envelope polyhedra), as both methods share the same theoretical basis, differing only in their way of application. In the Extreme Stress Method, the parametric equations for the response quantities are used (with normal stress σ as parameter), while in the Gupta-envelope Method the parameter σ is eliminated and the resulting response envelope is used instead (see Anastassiadis 1993, pp. 95 and 96, for the isotropic case). The approximate Extreme Force Method is also relatively reliable (see Panetsos and Anastassiadis 1998) and, most important, its application is simple and effortless.

Finally, a numerical example illustrating the general case of a nonsymmetric multistory building under seismic excitation defined by *different* spectral shapes along the principal excitation axes helps to clarify and confirm the presented theoretical results.

NOTATION

In the present paper, for reasons that will become obvious in the next section, two types of seismic ground motions relative to the principal axes system Opwv are used: the *actual* seismic motion and the *transposed* seismic motion. These motions include only the two horizontal earthquake components, described by spectra S^a and S^b. The vertical earthquake component (described by spectrum S^c) is ignored, because its effects on

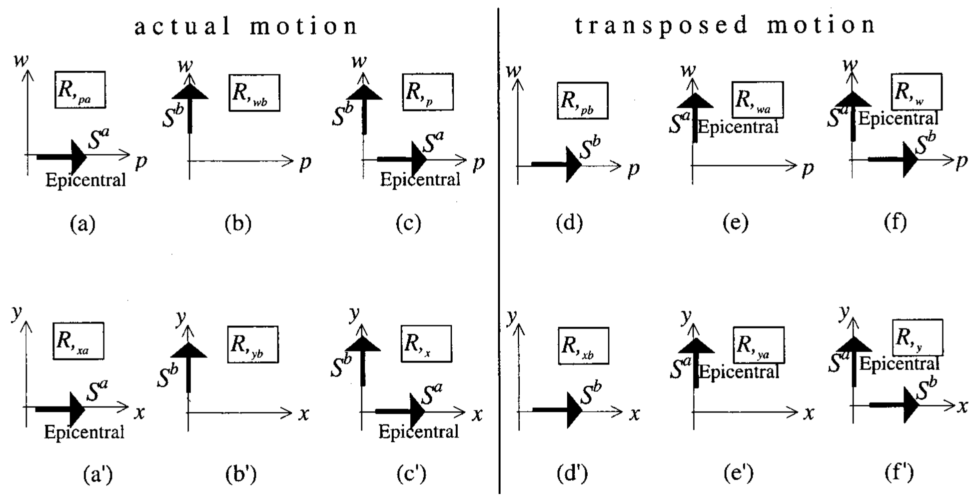

Figure 1. Response quantity notation for unidirectional (a, b, d, e, a′, b′, d′, e′) and bidirectional (c, f, c′, f′) seismic excitations.

the structure can be studied separately and can then be combined with the effects due to the horizontal components by the well-known SRSS combination rule.

ACTUAL SEISMIC MOTION

In this case, the design spectra S^a (epicentral) and S^b (lateral) are applied either individually (unidirectional inputs) or simultaneously (bidirectional inputs) in the directions of the principal axes p (epicentral) and w (lateral), as defined in the Introduction (Figures 1a, b, c). The peak value of a typical response quantity R (force or displacement) is denoted as

- $R_{,pa}$ and $R_{,wb}$ for *unidirectional* inputs, where the first index after the comma refers to the direction of the seismic motion (p or w) and the second (a or b) to the corresponding motion's spectrum (S^a or S^b) (Figures 1a and b), and as
- $R_{,p}$ for *bidirectional* inputs, where the single index after the comma refers to the direction in which the epicentral spectrum S^a is applied (Figure 1c).

Indices in front of the comma characterize the response quantity itself, while indices (i,j) denote the number of the vibration mode to which the response quantity belongs (e.g., $M_{xi,pa}$, $M_{yj,pa}$, $M_{xi,wb}$, $M_{yj,wb}$ or $M_{x,p}$, $M_{y,p}$, etc.). It is clear that the comma separates indices referring to the seismic input (after the comma) to "nonseismic" indices (before the comma).

TRANSPOSED SEISMIC MOTION

In this case, the roles of spectra S^a and S^b are interchanged: The epicentral spectrum S^a is applied in the direction of the lateral principal axis w, while the lateral spectrum S^b is applied in the direction of the epicentral principal axis p (Figures 1d, e, f). The meaning of the indices remains unchanged. Thus, for bidirectional seismic input the corre-

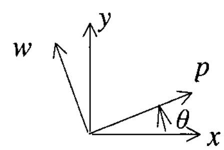

Figure 2. Definition of angle θ.

sponding peak value of a response quantity R is symbolized by $R_{,w}$, where the single index w after comma refers, as in the case of actual seismic motion, to the direction in which the epicentral spectrum S^a is applied (Figure 1f). Finally, in Figures 1a′–f′ the aforementioned notation is appropriately modified so as to refer to the fixed reference system Oxyz of the building structure.

TENSOR TRANSFORMATION OF RESPONSE QUANTITIES AND CRITICAL DIRECTION

We assume that the epicentral principal axis p of the ground motion is defined in terms of an angle θ relative to the x axis of the fixed reference system of the structure (Figure 2).

If S^a is the design spectrum in the direction of p axis and S^b the design spectrum in the direction of w axis (*actual* seismic motion), the *actual* peak value of a response quantity R is the following (Der Kiureghian 1981, Wilson et al. 1981):

$$R_{,p}{}^2 = R_{,pa}{}^2 + R_{,wb}{}^2 = \sum_i \sum_j \varepsilon_{ij}(R_{i,pa}R_{j,pa} + R_{i,wb}R_{j,wb}) \tag{1a}$$

In the above expression, $\varepsilon_{i,j}$ denotes the correlation coefficient between the responses in modes i and j, and $R_{i,pa}$ and $R_{i,wb}$ denote the modal values of quantity R corresponding to the excitations defined by the indices after comma. If S^b is the design spectrum in the direction of the p axis and S^a is the design spectrum in the direction of the w axis (*transposed* seismic motion), then we obtain the *transposed* peak value of R as

$$R_{,w}{}^2 = R_{,pb}{}^2 + R_{,wa}{}^2 = \sum_i \sum_j \varepsilon_{i,j}(R_{i,pb}R_{j,pb} + R_{i,wa}R_{j,wa}) \tag{1b}$$

and the correlation term

$$R_{pw} = R_{pw,a} - R_{pw,b} = \sum_i \sum_j \varepsilon_{i,j}(R_{i,pa}R_{j,wa} - R_{i,pb}R_{j,wb}) \tag{1c}$$

The modal values $R_{i,p}$ and $R_{i,w}$ are connected to the modal values $R_{i,x}$ and $R_{i,y}$ through the following relations, which are independent of the used earthquake spectrum S^a or S^b:

$$R_{i,p} = +R_{i,x}\cos\theta + R_{i,y}\sin\theta \tag{2a}$$

$$R_{i,w} = -R_{i,x}\sin\theta + R_{i,y}\cos\theta \tag{2b}$$

Inserting these relations in the right-hand terms of Equations 1a, 1b, and 1c, we obtain (see Anastassiadis 1993):

$$R_{,p}{}^2 = R_{,x}{}^2 \cos^2\theta + R_{,y}{}^2 \sin^2\theta + R_{xy} \sin 2\theta \tag{3a}$$

$$R_{,w}{}^2 = R_{,x}{}^2 \sin^2\theta + R_{,y}{}^2 \cos^2\theta - R_{xy} \sin 2\theta \tag{3b}$$

$$R_{pw} = -(1/2)(R_{,x}{}^2 - R_{,y}{}^2)\sin 2\theta + R_{xy} \cos 2\theta \tag{3c}$$

where

$$R_{,x}{}^2 = R_{,xa}{}^2 + R_{,yb}{}^2 = \sum_i \sum_j \varepsilon_{i,j}(R_{i,xa}R_{j,xa} + R_{i,yb}R_{j,yb}) \tag{4a}$$

$$R_{,y}{}^2 = R_{,xb}{}^2 + R_{,ya}{}^2 = \sum_i \sum_j \varepsilon_{i,j}\ (R_{i,xb}R_{j,xb} + R_{i,ya}R_{j,ya}) \tag{4b}$$

$$R_{xy} = R_{xy,a} - R_{xy,b} = \sum_i \sum_j \varepsilon_{i,j}\ (R_{i,xa}R_{j,ya} - R_{i,xb}R_{j,yb}) \tag{4c}$$

It is important to note that Equations 3a, 3b, and 3c are similar to the transformation rules for the components of a symmetric second-order tensor. Consequently, the four quantities $R_{,x}{}^2$, $R_{,y}{}^2$, R_{xy}, and R_{yx} can be considered as components of a symmetric second-order tensor $\mathcal{R}$, represented analytically by matrices:

$$\mathbf{R}_0 = \begin{bmatrix} R_{,x}{}^2 & R_{xy} \\ R_{yx} & R_{,y}{}^2 \end{bmatrix}$$

and

$$\mathbf{R}_\theta = \begin{bmatrix} R_{,p}{}^2 & R_{pw} \\ R_{wp} & R_{,w}{}^2 \end{bmatrix}$$

in the *Oxy* and *Opw* reference systems, respectively. This tensor is similar to the well-known plane stress tensor:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

which is familiar to any engineer from elementary Mechanics of Materials. Due to its tensorial character, the arbitrary response quantity R is characterized by the following properties that are common to all symmetric second-order tensors:

(a) The trace and the determinant of the above matrices do not depend on the orientation of the earthquake excitation:

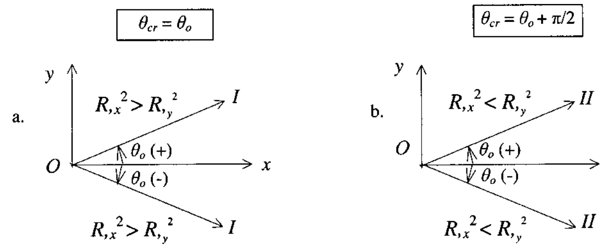

Figure 3. Determination of unfavorable seismic directions.

$$R_{,x}^{\ 2}+R_{,y}^{\ 2}=R_{,p}^{\ 2}+R_{,w}^{\ 2} \tag{5a}$$

and

$$R_{,x}^{\ 2}R_{,y}^{\ 2}-R_{xy}^{\ 2}=R_{,p}^{\ 2}R_{,w}^{\ 2}-R_{pw}^{\ 2}. \tag{5b}$$

(b) There is a specific earthquake orientation defined by the axes (I, II) for which the correlation term R_{pw} vanishes. This specific orientation is defined by an angle $\theta_{cr}=\theta_o$ or $\theta_{cr}=\theta_o+\pi/2$, where θ_o the acute angle $(-45° \leqslant \theta_0 \leqslant +45°)$:

$$\theta_o=(1/2)\tan^{-1}[2R_{xy}/(R_{,x}^{\ 2}-R_{,y}^{\ 2})], \tag{6}$$

according to Equation 3c and the indications in Figure 3.

For the bidirectional earthquake of Figure 4a, as well as for its transposition in Figure 4b, the response quantity R^2 takes the following maximum and minimum values, respectively:

$$\max R^2=R_{,I}^{\ 2}=(R_{,x}^{\ 2}+R_{,y}^{\ 2})/2 + \sqrt{[(R_{,x}^{\ 2}-R_{,y}^{\ 2})/2]^2+R_{xy}^{\ 2}} \tag{7a}$$

$$\min R^2=R_{,II}^{\ 2}=(R_{,x}^{\ 2}+R_{,y}^{\ 2})/2 - \sqrt{[(R_{,x}^{\ 2}-R_{,y}^{\ 2})/2]^2+R_{xy}^{\ 2}} \tag{7b}$$

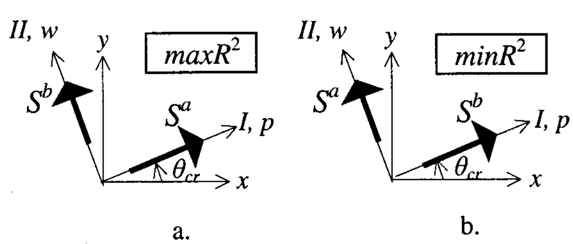

Figure 4. Favorable and unfavorable seismic directions.

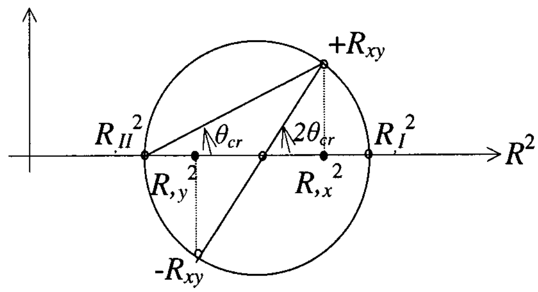

Figure 5. Graphical calculation of response extreme values using Mohr's circle.

The form of these relations allows for a graphical calculation of $R_{,I}^2$, $R_{,II}^2$ and angle θ_{cr} using Mohr's circle (Figure 5).

(c) The correlation term R_{pw} takes its maximum value:

$$\max R_{pw} = R_{12} = (1/2)(R_{,I}^2 - R_{,II}^2) \tag{8}$$

for a seismic excitation along axes 1 and 2, which are defined by the bisectors of the angles formed by the axes I and II. For these seismic directions, a response quantity R takes the value:

$$R_{,1}^2 = R_{,2}^2 = (1/2)(R_{,I}^2 + R_{,II}^2), \tag{9}$$

i.e., the interchange of the input spectra S^a and S^b along the axes 1 and 2 does not affect the peak value of R.

It is clear from the preceding considerations that the calculation of the maximum and minimum values of an arbitrary response quantity requires four independent dynamic analyses of the structure, applying input spectra S^a and S^b as shown in Figures 1a′–b′ and 1d′–e′. All necessary terms, e.g., the modal values in the right-hand sides of Equations 4a, 4b, and 4c, are routinely calculated by current standard linear dynamic analysis programs. Then, using Equations 4a, 4b, and 4c, $R_{,x}^2$, $R_{,y}^2$, and R_{xy} can be computed, and from Equations 7a and 7b the maximum and minimum values of any response quantity R can be immediately obtained, with no need to previously calculate the critical angle θ_{cr}. Finally, the contribution of the vertical seismic component is to be added to the above values, according to the SRSS combination rule. It is obvious that all mentioned relations can be easily implemented in current standard software for multicomponent seismic analysis.

SPECIAL CASES

ISOTROPIC SEISMIC EXCITATION ($S^a = S^b$)

In this case:

$$R_{,x} = R_{,y} \quad \text{and} \quad R_{,p} = R_{,w}$$

and, according to Equation 5a,

$$R_{,x}^{2}=R_{,p}^{2}, \tag{10}$$

that is, the value of R does not depend on the earthquake input angle θ.

UNIDIRECTIONAL SEISMIC EXCITATION ($S^{b}=0$)

In this case:

$$R_{,yb}=0 \ \text{and} \ R_{,xb}=0$$

and Equations 4a, 4b, and 4c can now be written as follows:

$$R_{,x}^{2}=R_{,xa}^{2}=\sum_{i}\sum_{j}\varepsilon_{i,j}R_{i,xa}R_{j,xa} \tag{11a}$$

$$R_{,y}^{2}=R_{,ya}^{2}=\sum_{i}\sum_{j}\varepsilon_{i,j}R_{i,ya}R_{j,ya} \tag{11b}$$

$$R_{xy}=R_{xy,a}=\sum_{i}\sum_{j}\varepsilon_{i,j}R_{i,xa}R_{j,ya} \tag{11c}$$

By inserting these values for $R_{,x}^{2}$, $R_{,y}^{2}$ and R_{xy} in Equations 6 and 7, the critical direction as well as the maximum and minimum response values can be calculated.

BIDIRECTIONAL EXCITATION USING ANALOGOUS SPECTRA ($S^{b}=\lambda S^{a}$)

In this case (loading ratio $0<\lambda<1$):

$$R_{,yb}=\lambda R_{,ya}$$

$$R_{,xb}=\lambda R_{,xa}$$

$$R_{xy,b}=\lambda^{2} R_{xy,a}$$

Equations 4a, 4b, and 4c can now be written as

$$R_{,x}^{2}=R_{,xa}^{2}+\lambda^{2}R_{,ya}^{2} \tag{12a}$$

$$R_{,y}^{2}=\lambda^{2}R_{,xa}^{2}+R_{,ya}^{2} \tag{12b}$$

$$R_{xy}=(1-\lambda^{2})R_{xy,a} \tag{12c}$$

Similarly, Equation 6 gives

$$\theta_{0}=(1/2)\tan^{-1}[2R_{xy,a}/(R_{,xa}^{2}-R_{,ya}^{2})] \tag{13}$$

Furthermore, according to Equations 7a and 7b, the maximum and minimum responses are

$$\max R^{2}=(1+\lambda^{2})(R_{,xa}^{2}+R_{,ya}^{2})/2+(1-\lambda^{2})\sqrt{(R_{,xa}^{2}-R_{,ya}^{2})/2]^{2}+R_{xy,a}^{2}} \tag{14}$$

$$\min R^2 = (1+\lambda^2)(R_{,xa}{}^2 + R_{,ya}{}^2)/2 - (1-\lambda^2)\sqrt{[(R_{,xa}{}^2 - R_{,ya}{}^2)/2]^2 + R_{xy,a}{}^2} \qquad (15)$$

It is apparent that the critical earthquake input angle θ_{cr} is not a function of the loading ratio λ. However, it is important to note that the maximum response quantity max $R2$ does in fact strongly depend on the loading ratio λ.

DESIGN FORCES

In general, the design of a structural member cannot be based only on the extreme value *exR* of a single stress resultant (or internal force) *R*. In most cases, especially in reinforced concrete structures, the knowledge of the *most unfavorable* combination of two or even three *simultaneous* internal forces at a specified cross section (briefly, "sectional forces") of a structural element is required. The use of the extreme values of the relevant sectional forces, which, in general, do not occur simultaneously, leads, for the most part, to unnecessary over-dimensioning.

In seismic design codes this situation is usually dealt with by allowing the empirical "percentage combination rules" of the 100-30-30 or 100-40-40 type, for which no theoretical evidence is available (Wilson et al. 1995; Menun and Der Kiureghian 1998, 2000a). In contrast to this common practice, exact methods in the case of isotropic seismic excitation (Gupta and Singh 1977, Gupta and Chu 1977) as well as in the case of isotropic or orthotropic excitation (Anastassiadis 1993) are not widely known and, despite their advantages, have not yet been implemented in structural analysis software packages.

Here, two recently developed methods are presented, which can be used for the determination of "unfavorable combinations" of probable simultaneous sectional forces in case of an orthotropic seismic excitation. In the case of isotropic seismic action, see Anastassiadis 1993, Anastassiadis and Avramidis 1996, and Panetsos and Anastassiadis 1998.

THE EXTREME STRESS METHOD

In general, the analysis of R/C structures, whether seismic or not, is carried out in two clearly separated phases:

- In the first phase, the sectional forces (or internal forces, i.e., bending moments, axial forces, etc.) at all relevant member sections are calculated, assuming that the structure's behavior is linear-elastic and the R/C member sections homogeneous.
- In the second phase, the design (proportioning, sizing) of the structural members is carried out, based on the sectional forces determined in the first phase and taking into account cracking of the sections as well as nonlinear material properties.

The determination of the "unfavorable combinations" of the relevant sectional forces in the structure pertains to the first phase. Consequently, it is legitimate to provisionally calculate normal and shear stresses at a specified section as well as the corresponding bending moments, shear forces and axial forces according to the rules known from elementary Mechanics of Materials (i.e., assuming homogeneous sections). Thus, for example, at any point in a rectangular column section, it is possible to provisionally determine the axial stress σ corresponding to a prescribed probable simultaneous combination of sectional forces M_{ξ}, M_{η}, N that "produce" stress σ. Now, if σ is chosen to be the extreme stress $ex\sigma_k$ (with positive or negative sign) at one of the corners of the rectangular section, the corresponding combination of sectional forces $M_{\xi k}$, $M_{\eta k}$, N_k is a probable "unfavorable combination" of simultaneous sectional forces, because of the very fact that this combination "produces" an extreme stress in the section. For the four corners $k=1, 2, 3, 4$, a total number of $(2\times4)=8$ unfavorable combinations (four for positive-signed $ex\sigma_k$ and four for negative-signed $ex\sigma_k$) results, which cover all possible cases of extreme stresses to be generated in a rectangular section.

Based on the above idea, the *extreme stress method* can be applied in case of *orthotropic* seismic excitation as follows: First, using Equations 7a and 7b, the extreme normal stresses

$$ex\sigma_k=\pm\sqrt{\max\sigma_k^2},\ \ k=1,2,\ldots \tag{16}$$

and their corresponding critical directions $\theta_{cr,k}$ are calculated at the corners of the member section under consideration. For the now known unfavorable orientation of the orthotropic seismic excitation (Figure 4a), the probable simultaneous (to $ex\sigma_k$) value of a sectional force S is determined from (Gupta and Chu 1977):

$$S_{\sigma_k}=C_{\sigma_k s}/ex\sigma_k \tag{17}$$

where:

$$C_{\sigma_k s}=\sum_i\sum_j\varepsilon_{i,j}(\sigma_{ki,Ia}S_{j,Ia}+\sigma_{ki,IIb}S_{j,IIb}) \tag{18}$$

In this relation, σ_{ki} and S_j ($i,j=1,2,...,N$) denote the modal values of σ_k and S for seismic excitation indicated by the indexes after comma. After transformation of these modal values according to Equations 2a and 2b, the correlation coefficient $C_{\sigma_k s}$ can be written as:

$$C_{\sigma_k s}=C_{\sigma_k s,x}\cos^2\theta_{cr,k}+C_{\sigma_k s,y}\sin^2\theta_{cr,k}+C_{\sigma_k s,xy}\sin 2\theta_{cr,k} \tag{19}$$

where:

$$C_{\sigma_k s,x}=\sum_i\sum_j\varepsilon_{i,j}\,(\sigma_{ki,xa}S_{j,xa}+\sigma_{ki,yb}S_{j,yb}) \tag{20a}$$

$$C_{\sigma_k s,y}=\sum_i\sum_j\varepsilon_{i,j}\,(\sigma_{ki,xb}S_{j,xb}+\sigma_{ki,ya}S_{j,ya}) \tag{20b}$$

$$C_{\sigma_k s,xy}=(1/2)\left[\sum_i\sum_j\varepsilon_{i,j}\,(\sigma_{ki,xa}S_{j,ya}+\sigma_{ki,ya}S_{j,xa})-\sum_i\sum_j\varepsilon_{i,j}\,(\sigma_{ki,xb}S_{j,yb}+\sigma_{ki,yb}S_{j,xb})\right] \tag{20c}$$

In the above equations, the modal values of σ_k and S for the seismic excitations shown in Figures 1a′–b′ and 1d′–e′ are used. For a given $\theta_{cr,k}$, the correlation factor $C_{\sigma_k s}$ is calculated from Equation 19. Then, the probable simultaneous (to $ex\sigma_k$) value of sectional force S follows from Equation 17. This calculation procedure is repeated for all the other sectional forces that must be taken into account for the section's design (bending moments, normal forces, etc.). The set of all sectional forces $S_1, S_2, \ldots$ concurrent to $ex\sigma_k$ define a specific point on the Gupta-envelope in the corresponding 2-D or 3-D response space. Finally, in the same way, the "unfavorable combinations" of sectional forces are determined, which correspond to extreme shear stresses $ex\tau_k$ in the section.

THE APPROXIMATE EXTREME FORCE METHOD

This method is based on the following, relatively reliable, rule: *Each unfavorable combination of two or more sectional (or internal) forces at a structural member's section comprises the extreme value of one of these forces and the probable simultaneous values of the others*. The extreme values of any given sectional force R are (see Equation 7a):

$$exR = \pm\sqrt{\max R^2} \tag{21}$$

and the corresponding critical orientation angle $\theta_{cr,r}$ can be calculated from Equation 6. For a seismic excitation according to Figure 4a, the probable simultaneous (to exR) value of any other sectional force S is (Gupta and Chu 1977):

$$S_R = C_{rs}/exR \tag{22}$$

where

$$C_{rs} = \sum_i \sum_j \varepsilon_{i,j}(R_{i,Ia}S_{j,Ia} + R_{i,IIb}S_{j,IIb}) \tag{23}$$

In this expression of the correlation factors C_{rs}, R_i and S_j ($i,j = 1,2,\ldots,N$) denote the modal values of sectional forces R and S for seismic excitations as indicated by the indexes after comma. After transformation of these modal values according to Equations 2a and 2b, the correlation coefficient C_{rs} can be written as:

$$C_{rs} = C_{rs,x}\cos^2\theta_{cr,r} + C_{rs,y}\sin^2\theta_{cr,r} + C_{rs,xy}\sin 2\theta_{cr,r} \tag{24}$$

where

$$C_{rs,x} = \sum_i \sum_j \varepsilon_{i,j}(R_{i,xa}S_{j,xa} + R_{i,yb}S_{j,yb}) \tag{25a}$$

$$C_{rs,y} = \sum_i \sum_j \varepsilon_{i,j}(R_{i,xb}S_{j,xb} + R_{i,ya}S_{j,ya}) \tag{25b}$$

$$C_{rs,xy} = (1/2)\left[\sum_i \sum_j \varepsilon_{i,j}(R_{i,xa}S_{j,ya} + R_{i,ya}S_{j,ya}) - \sum_i \sum_j \varepsilon_{i,j}(R_{i,xb}S_{j,yb} + R_{i,yb}S_{j,xb})\right] \tag{25c}$$

In the above equations, the modal values of R and S for the seismic excitations shown in Figures 1a′–b′ and 1d′–e′ are used. For a given $\theta_{cr,k}$, the correlation factor C_{rs} is calculated from Equation 24. Then, the probable simultaneous (to exR) value of sectional force S follows from Equation 22. This calculation procedure is repeated for all the other sectional forces that must be taken into account for the section's design (bending moments, normal forces, etc.). The set of all sectional forces (exR included) concurrent to exR define a specific point on the Gupta-envelope in the corresponding 2-D or 3-D response space. As an example, in the familiar case of biaxial bending of an axially stressed element, the following six combinations of probable simultaneous sectional forces arises for the element under consideration:

1.	$exM_\xi(+)$,	$M_{\eta,1}$,	N_1	4.	$exM_\xi(-)$,	$-M_{\eta,1}$,	$-N_1$
2.	$M_{\xi,2}$,	$exM_\eta(+)$,	N_2	5.	$-M_{\xi,2}$,	$exM_\eta(-)$,	$-N_2$
3.	$M_{\xi,3}$,	$M_{\eta,3}$,	$exN(+)$	6.	$-M_{\xi,3}$,	$-M_{\eta,3}$,	$exN(-)$

The extreme values exM_ξ, exM_η, exN can be calculated using Equation 21 while the corresponding concurrent forces for each combination are determined from Equation 22.

NUMERICAL EXAMPLE

The theoretical results of the foregoing sections will be applied to a multistory building consisting of six similar stories with a total height $H=6\times3.0$ m=18.0 m (Figure 6).

Each story has a mass $m=79.46$t and a mass moment of inertia $J_z=1086$tm^2. The strong seismic motion in the p-axis' direction and the orthogonal seismic excitation in the w-axis' direction are defined by the following design response spectra, respectively:

$$S^a(T)=A^aR^a(T)/q$$

and

$$S^b(T)=A^bR^b(T)/q$$

where $A^a=0.16$g and $A^b=0.12$g are the peak ground accelerations, $q=2$ is the behavior factor, and $R^a(T)$, $R^b(T)$ denote the spectra shapes shown in Figure 6.

Using these numerical data, four independent response spectrum analyses in the directions of the building's axes x and y have been carried out, according to the four seismic excitations defined in Figures 1a′–b′ and 1d′–e′. For these analyses the well-known computer program SuperEtabs is used. A damping ratio $\xi=5\%$ and the seven leading mode shapes, with periods equal to 0.775, 0.690, 0.382, 0.251, 0.216, 0.145, 0.122 sec, respectively, are taken into account. For each seismic excitation, the modal values σ_{ki} of the normal stresses σ_k ($k=1, 2, 3, 4$) at the four corners of the fixed-end section (base) of column C_3 are calculated. Furthermore, the modal values $M_{\xi i}$, $N_{\zeta i}$, $M_{\eta i}$ of the internal forces M_ξ, N_ζ, M_η at the same section are determined.

APPLICATION OF THE EXTREME STRESS METHOD

First, by using Equations 6, 7, and 16, the magnitudes of extreme stresses exσ_k and the corresponding critical angles θ_{cr}, defining the orientation of epicentral axis p, have

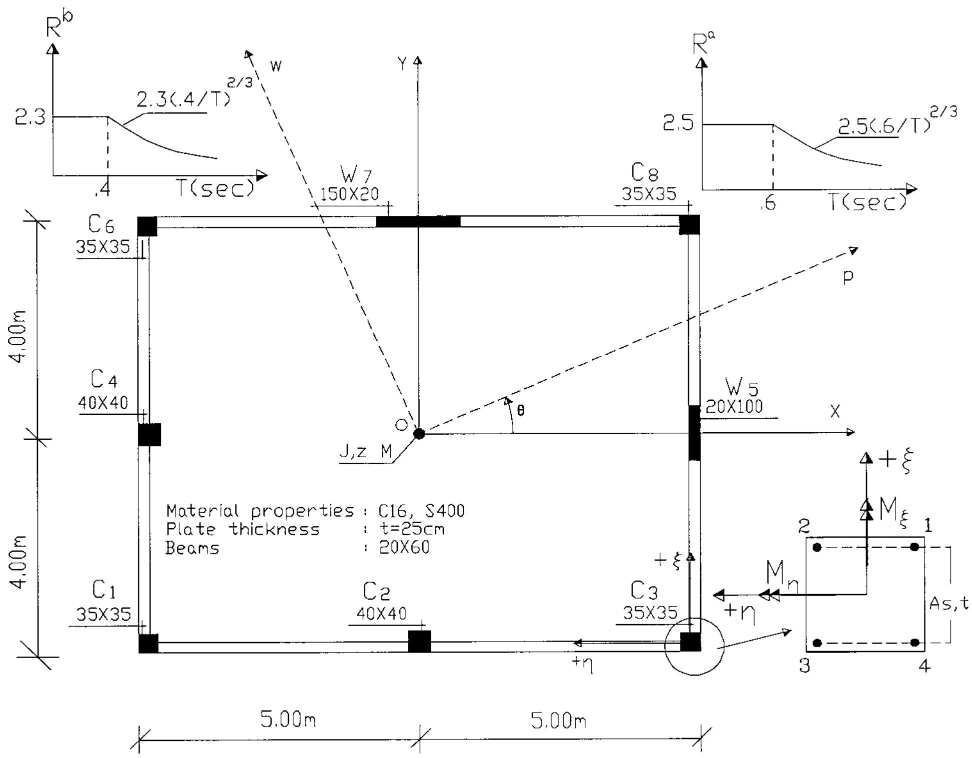

Figure 6. Typical floor plan of a 6-story building and design response spectra.

been calculated. The results are given in Table 1 along with the "unfavorable combinations" of stress resultants M_ξ, N_ζ, M_η. The latter have been determined by applying Equation 17 and correspond to the positive extreme values exσ_k. (For the negative extreme values, the same combinations with opposite signs are valid).

Then, by adding stress resultants $M_{\xi,g}=-5.97$ kNm, $N_{\zeta,g}=-569.13$ kN and $M_{\eta,g}=+2.69$ kNm due to vertical, i.e., gravity, loads (G+0.3Q) to the values in Table 1, the final unfavorable design combinations of internal forces are obtained (Table 2).

Table 1. Unfavorable combinations of seismic action effects

k	ex$\sigma_k(+)$ [kN/m^2]	$\theta_{cr,k}$ [°]	M_ξ [kNm]	N_ζ [kN]	M_η [kNm]
1	19267.83	18.34	+85.63	−30.92	+53.83
2	13577.46	120.00	−75.95	−430.97	+46.20
3	21401.55	28.97	−75.62	+109.17	−63.52
4	20799.62	117.17	−50.77	+449.79	+71.63

Table 2. Unfavorable design combinations

k	$\mathrm{ex}\sigma_k$	M_ξ [kNm]	N_ζ [kN]	M_η [kNm]	A_{tot} [cm^2]	A_{tot}* [cm^2]
1	$\mathrm{ex}\sigma_1(+)$	+79.66	−600.05	+56.52	12.46	18.84
2	$\mathrm{ex}\sigma_2(+)$	−81.92	1000.10	+48.89	14.21	19.14
3	$\mathrm{ex}\sigma_3(+)$	−81.59	−459.96	−60.83	14.14	18.55
4	$\mathrm{ex}\sigma_4(+)$	−56.74	−119.34	+74.32	**15.34**	**19.16**
1	$\mathrm{ex}\sigma_1(-)$	−91.59	−538.21	−51.14	14.63	18.84
2	$\mathrm{ex}\sigma_2(-)$	+69.98	−138.16	−43.51	12.72	19.14
3	$\mathrm{ex}\sigma_3(-)$	+69.65	−678.29	+66.21	11.62	18.55
4	$\mathrm{ex}\sigma_4(-)$	+44.80	1018.91	−68.94	11.09	**19.16**

The maximum reinforcement area $A_{tot}=15.34$ cm^2 results from the fourth combination in Table 2. Last column in Table 2 shows the reinforcement A_{tot}* corresponding to the *rectangular* envelope. Obviously, A_{tot}* is always greater than A_{tot}. For the critical combination of the example under consideration, this increase reaches 25%.

APPLICATION OF THE EXTREME FORCE METHOD

First, by using Equations 6, 7, and 21, the magnitudes of extreme internal (or sectional) forces $\mathrm{ex}M_\xi$, $\mathrm{ex}M_\zeta$, $\mathrm{ex}M_\eta$ and the corresponding critical angles θ_{cr}, defining the orientation of epicentral axis p, have been calculated. The results are given in Table 3 along with the values of the other two sectional forces that are simultaneous to the positive extreme magnitudes $\mathrm{ex}M_\xi(+)$, $\mathrm{ex}M_\zeta(+)$, $\mathrm{ex}M_\eta(+)$, and have been calculated to Equation 22.

Then, by adding the sectional forces due to gravity loads to the values in Table 3, the final unfavorable design combinations are obtained (Table 4).

The maximum reinforcement area $A_{tot}=12.88$ cm^2 results from the fifth combination in Table 4 and is smaller than the maximum reinforcement area $A_{tot}=15.34$ cm^2 calculated according to the extreme stress method (see Table 2). Last column in Table 4 shows the reinforcement A_{tot}* corresponding to the *rectangular* envelope. Obviously, A_{tot}* is always greater than A_{tot}. For the critical combination of the example under consideration, this increase reaches 54.5%.

Table 3. Unfavorable combinations of seismic action effects

k	extreme force	$\theta_{cr,i}$ [°]	M_ξ [kNm]	N_ζ [kN]	M_η [kNm]
1	$\mathrm{ex}M_\xi(+)$	−11.27	+93.14	+168.28	+17.34
2	$\mathrm{ex}N_\zeta(+)$	+110.40	+60.66	+463.10	−56.61
3	$\mathrm{ex}M_\eta(+)$	+52.03	+32.38	−230.87	+81.81

Table 4. Unfavorable design combinations

k	extreme force	M_ξ [kNm]	N_ζ [kN]	M_η [kNm]	A_{tot} [cm^2]	A_{tot}* [cm^2]
1	ex$M_\xi(+)$	+87.16	−400.85	+20.03	10.60	19.36
2	ex$N_\zeta(+)$	+54.69	−106.03	−53.92	11.43	19.60
3	ex$M_\eta(+)$	+26.41	−800.00	+84.50	10.39	**19.90**
4	ex$M_\xi(-)$	−99.11	−737.41	−14.65	12.05	19.36
5	ex$N_\zeta(-)$	−66.63	−1032.23	+59.30	**12.88**	19.60
6	ex$M_\eta(-)$	−38.35	−338.26	−79.12	11.17	**19.90**

In general, as in case of isotropic seismic excitation (see Panetsos and Anastassiadis 1998), the extreme force method turns up to be less unfavorable then the extreme stress method.

CONCLUSIONS

In this paper, a general solution for the three-component orthotropic seismic excitation problem is presented. It offers, within the framework of response spectrum analysis, a rational procedure for determining the maximum and minimum values of any typical response quantity R of a structure along with the simultaneous values of other related quantities that are relevant for the structural members' design in order to avoid unnecessary over-dimensioning. It also provides a simple means of determining the critical orientation θ_{cr} associated with the extreme values of R. Thus, knowledge of the orientation of the principal axes is not required beforehand. In contrast to the SRSS rule prescribed by many design codes, the presented method can explicitly account for the correlation of the different seismic components by incorporating in its formulation the Penzien-Watabe model of ground motion. This fact makes it particularly useful in the dynamic analysis of curved bridges. It is shown that for equal horizontal design spectra, the structure's response quantities due to bidirectional seismic excitation are not a function of the earthquake's orientation. This is not the case if empirical rules without theoretical justification are used. Finally, two methods are presented, which allow for the determination of the most unfavorable combinations of two or more internal (or sectional) forces acting concurrently at a structural member's section (e.g., two bending moments and an axial force at a column's base). All given relations are of a computationally simple form and can be easily implemented in current standard dynamic analysis software. A numerical example clarifies and confirms the presented theoretical results.

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